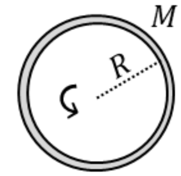
APPC, Mechanics: Unit γ HW 3

Name: ______________________
Hr: ___ Due at beg of hr on: __________

Uγ, HW3, P1

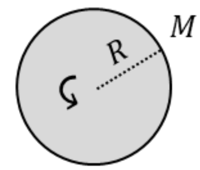
Reference Videos: (1) "Derivation of the Rotational Inertia of a Solid Disk"
(2) "Rotational Inertia for a Cylinder" YouTube, lasseviren1



A. Write the equation for the moment of inertia of a hoop (or ring) for rotating, wheel-like, about its central axis. See the figure at right.

B. An equation that ALWAYS WORKS to find any moment of inertia is your answer to Part D of HW2, P5. Rewrite that equation here.

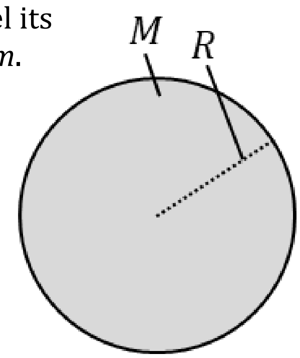
C. Explain how your Part B answer easily simplifies to your Part A answer. In your response, specifically mention what is true for each of the many *dm* pieces of the hoop.



D. Write the equation for the moment of inertia of a solid disk of uniform mass density for rotating, wheel-like, about its central axis. See the figure at right.

We now will derive your Part D answer. To do so, we must break the disk into many very-thin hoops.

E. Into the large figure of the solid disk at right, draw ONE of the very-thin hoops. Label its distance from the axis as *r*, its very-thin thickness as *dr*, and its very-tiny mass as *dm*.



In the previous assignment, we met the linear mass density λ . Disks, however, don't have *lengths*; they have *areas*. This brings us to **surface mass density** $\sigma = \frac{mass}{area}$.

F. Slight digression: Using the equation just presented, write the expression for the σ of the entire disk, in terms of *M* and *R*.

G. But now, we need an expression for σ for our very-thin hoop, and to do that, we need its mass. What is that mass? (HINT: See Part E.)

H. We also need the hoop's area. If we took the hoop out, cut it as if snipping a rubber band, and unrolled it, it would be a rectangle having length _________ and width ___; thus, its area would be ___________.

I. Substitute your Parts G and H answers into the σ equation given above Part F, to obtain an expression for σ for our very-thin hoop.

J. Solve your Part I equation for *dm*.

K. Substitute your Part J answer into your Part B answer, and simplify. Check your drawing above, figure out the integration limits for *r*, and be sure to include those limits on your answer.

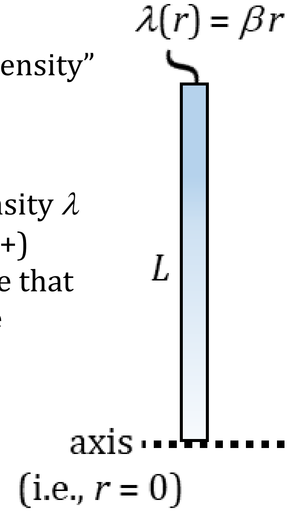
L. Carry out the integration of your Part K answer.

M. To finish, substitute your Part F answer into your Part L answer. If you've done it correctly, this should yield your response from way back in Part D.

N. In the second video, the narrator derives the moment of inertia for a uniform cylinder. But a cylinder is nothing more than a very thick disk! Therefore, a cylinder's moment of inertia should be given by WHAT equation? Write it here.

Uγ, HW3, P2

Reference Videos: (1) "Rotational Inertia of a Slender Rod with Non-uniform Mass Density"
(2) "Rotational Inertia of a Disk with Non-uniform Mass Density"
YouTube, lasseviren1



First, we derive the moment of inertia of a NON-uniform rod having a linear mass density λ that varies with position r according to the known equation $\lambda(r) = \beta r$, where β is a (+) constant. (Perhaps you will notice the shading in the figure, which is meant to indicate that the rod becomes more dense as we approach the top of the page.) We wish to find the moment of inertia for the rod if it were to rotate about the axis shown at the bottom.

A. Start by writing the equation for finding any moment of inertia.
This was your answer to Part D of HW2, P5 and Part B of HW3, P1.

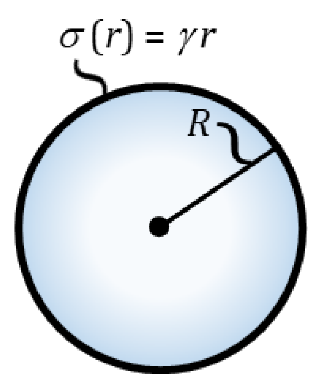
B. On the rod in the figure, draw in a tiny *dm* element, labeling it as *dm*. Designate that this *dm* element is a distance *r* from the axis AND that the *dm* element has a length *dr*.

C. We need the mass of the *dm* element from Part B. Since $\lambda = \frac{mass}{length}$, we can write $\lambda(r) = \frac{dm}{dr}$. Well, we are given that, for this NON-uniform rod, $\lambda(r) = \beta r$.
Combine these last two equations, and then solve your result for *dm*.

D. Substitute your Part C answer into your Part A answer. Include the limits of integration.

E. Integrate and simplify your Part D answer. You did it!

We now derive the I of a NON-uniform disk having a surface mass density $\sigma = \frac{mass}{area}$ that varies with distance from the axis r according to the known equation $\sigma(r) = \gamma r$, where γ is a (+) constant. The shading in the figure (assuming you can see it) indicates that the disk becomes more dense as we approach its edges. We wish to find the moment of inertia for the disk as it rotates about its central axis.



F. On the disk, draw in a tiny *dm* 'hoop' element, labeling it as *dm*.
Show that the element has a radius *r* AND that it has the thickness *dr*.

G. We need the mass of the *dm* 'hoop' element from Part F. Since $\sigma = \frac{mass}{area}$, we can also write $\sigma(r) = \frac{dm}{tiny-hoop\ area}$. Well, we know the left side of that equation, since $\sigma(r) = \gamma r$.
For the denominator on the right side....If we took the hoop out, cut it as if snipping a rubber band, and unrolled it, it would be a rectangle of length ________ and width ____; thus, its area would be ____________.

H. Combine the two $\sigma(r)$ equations in Part G AND the last equation in Part G...and then solve your result for *dm*.

I. Substitute your Part H answer into your Part A answer (which, of course, still applies, since it ALWAYS does...). Include the limits of integration.

J. Integrate and simplify your Part I answer. Done!

Uγ, HW3, P3
Reference Video: "The Parallel Axis Theorem"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

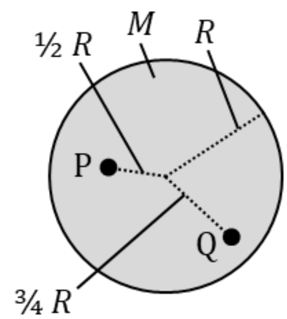
A. The **parallel-axis theorem** allows you to do what?

B. Write the equation for the parallel-axis theorem.

C. In your Part B answer, describe what is meant by the term that is squared. (Depending on the source, this term is given various labels, which is why I just refer to it as "the term that is squared").

D. Look at your Part B answer again and then CIRCLE your answers below.

"The rotational inertia through any non-*com* axis will always be LARGER SMALLER than the rotational inertia through a *com* axis. In other words, the rotational inertia through a *com* axis represents a MINIMUM MAXIMUM rotational inertia for any object about a given *com* axis."



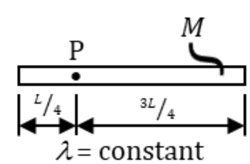
The figure at right shows a uniform disk that is to be rotated about a point that is NOT through its *com*. If you can imagine stabbing a cardboard pizza disk with a pencil at Point P (or, in just a minute, Point Q), and then holding the pencil and whirling the disk around on it...that's essentially what we're doing here.

E. Before we start, do you expect I to be larger about Point P, or about Point Q? Explain.

F. Okay, here we go. First, determine I_{com} for the disk.
HINT: Look back at Part D (and M) on HW3, P1.

G. Use the parallel-axis theorem to find I about Point P. Show work and fully simplify.

H. Now, determine the moment of inertia about Point Q. Show your work, simplify your answer, and then comment on your answers to Parts E, G, and H.



The figure shows a uniform rod being rotated about a point halfway between the center and one end. Rulers often have multiple, pre-drilled holes: If there were a hole at Point P and you put a pencil there and whirled the ruler around...that's what's happening.

I. Determine I_{com} for the rod. HINT: Look back in the commentary prior to Part E on HW2, P5.

J. Use the parallel-axis theorem to determine I for this situation. Show work and fully simplify.

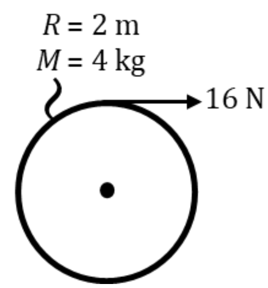
K. Look back at HW2, P5, Part H. How does your answer THERE compare with your Part J answer HERE?

Uγ, HW3, P4
Reference Video: "Rotational Dynamics"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

TRANSLATION ROTATION

A. At right, write equations for Newton's 2nd law. Either form (the YES-denominator form or the NO-denominator form) is fine.

The figure shows a uniform disk that is nailed to the wall through its *com* such that it can rotate about its *com*. A light string is wrapped around-and-around the disk, and a constant force is applied to the string. Assume that the nail is a frictionless axle.



B. Besides the tension in the string, there are two other forces that act on the disk. Into the figure, at the proper location(s), draw and label these two other forces.

C. In just a minute, you will apply Newton's 2nd law for rotation to the disk, BUT... when you do, you will NOT include the forces from your Part B answer. Why not?

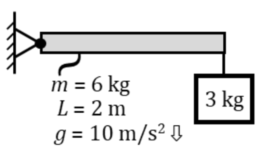
D. Determine the net torque on the disk. Include proper units.

E. Determine the I of the disk, about its *com*. HINT: See HW3, P3, Part F.

F. Use Newton's 2nd law and your Parts D and E answers to determine the angular acceleration of the disk.

G. Is your Part F answer constant for this situation, or not? Explain how you know.

H. If there WERE friction from the axle, what would that do to your Part F answer?



This figure shows a uniform rod that is pinned at one end and free at the other. At the free end, there is an additional suspended mass. The rod-mass system is released from rest, with the rod initially in a horizontal position.

I. Determine I for the rod. HINT: Begin with your work on Part I of HW3, P3 and then – because the axis is NOT through the *com* – use the parallel-axis theorem. (Or you could just look up the formula...)

J. Determine I for the suspended mass. Refer back to Parts A and C of HW2, P5.

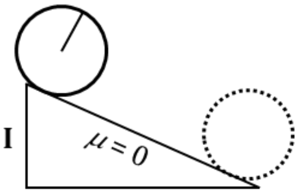
K. The total I for a compound system is simply the sum of the Is for each part. So, add your Parts I and J answers to obtain the total I of the rod-mass system.

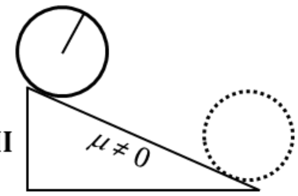
L. Determine the net initial torque on the rod-mass system.

M. Determine the initial angular acceleration of the rod-mass system.

N. Is your Part M answer constant, as the rod descends? Explain your answer.

Uγ, HW3, P5
Reference Video: "Rotational Dynamics (Part II)"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

I


II


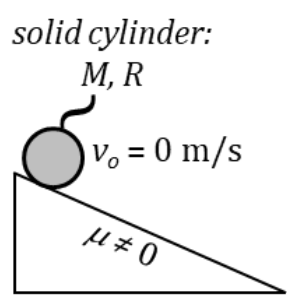
A. The cylinders at right start from rest. In Case I, there is no friction; in Case II, there IS. On the 'dashed' cylinders, use symbols to show what is happening in the 'dashed' location. These symbols include a labeled velocity arrow (originating on a cylinder's *com*) and pointing in the direction of the velocity (or not!) AND ↶ or ↷ to show rotation (or not!). *Don't represent anything that isn't happening*.

B. Notice the straight line 'painted' on the top-of-the-ramp cylinders. Onto each 'dashed' cylinder, draw an identical line...but at an orientation that is plausible, based on what you said in your Part A answers.

C. To reinforce what you've done in Parts A and B... "In Case I, the absence of friction means that the cylinder will merely _________ down the ramp, i.e., it will NOT _______ down the ramp. On the other hand, in Case II, the friction between the cylinder and ramp applies a net ________________ to the cylinder, which causes the cylinder to _______ down the ramp, rather than _________ down the ramp."

D. One VERY important point when dealing with a rolling object is that, in our FBD, we need to draw all forces where they _________________ __________. A second important point is that we should choose the axis to be through the object's ____________ ___ ________. When we do that, two forces that contribute ZERO torque to the rolling object are the ______________ force and the force of ______________. This is because the _________ ___ ____________ of those two forces go directly through the _______.

The figure shows a solid, uniform cylinder. When released from rest, it rolls without slipping. As we will see here, analyzing rolling requires us to use BOTH of Newton's 2nd laws (rotation and translation), as well as a bridge equation that connects the two. Three equations, three unknowns, yada-yada-yada...

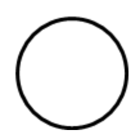


E. In the circle below the figure, draw the FBD that applies. HINT: Label the friction force merely as F_f. *Make NO reference to μ for the rest of this assignment.*

F. Write the Newton's 2nd law equation for rotation that applies, and simplify it. HINT: You might want to look back at HW3, P1, Part N.

G. Write the Newton's 2nd law equation for translation. There is only one: in the down-the-ramp direction.

H. Write the bridge equation that connects your Parts F and G answers. (See HW1, P5, Part A.)



I. Substitute your Part H answer into your Part G answer, and solve for the force of friction.

J. Substitute your Part I answer into your Part F answer, and solve for the angular acceleration.

TO BE CONTINUED...