APPC, Mechanics: Unit γ HW 4

Name: ____________________________________
Hr: ___ Due at beg of hr on: ____________

Uγ, HW4, P1

Reference Videos: (1) "Rotational Dynamics (Part II)"
(2) "Rotational Dynamics (Part III)"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

We begin by finishing the our work from HW3, P5….so you'll need to reference that now.

K. Substitute your Part J answer back into your bridge equation from Part H to obtain an expression for the translational acceleration of the cylinder.

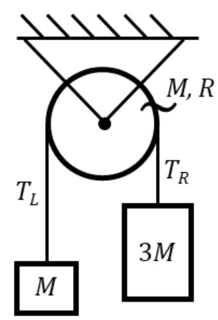
L. Substitute your Part J answer back into your Part F answer and simplify, to obtain an expression for the force of friction, in terms of the given quantities.

M. Show that another way to get your Part L answer is to substitute your Part K answer into your Part G answer.

N. Check that your expressions in Parts J-M yield the correct unit for each quantity you are after. After the units check out, put a ☺ in each blank.

Part J ___ Part L ___
Part K ___ Part M ___

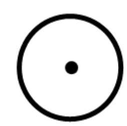
Moving on...



The figure shows a pulley system, with two masses attached. Unlike the Atwood's machines that we studied previously, <u>this pulley is NOT massless NOR frictionless</u>. This means that the pulley will have a nonzero moment of inertia that you must take into account. (Here, consider the pulley a uniform disk.) ALSO, the rope tensions on either side of the pulley will NOT be equal to each other. Let's begin...

A. Draw FBDs in the figures below. Label each force you draw. Use g to represent the acceleration due to gravity. (HINTS: The FBD for the pulley has four forces. Also, because friction is an internal force in the rope-pulley system, do NOT represent friction at this time.)

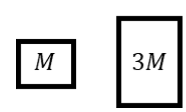
B. Write the equation for the moment of inertia of the pulley/uniform disk.



C. Write the Newton's 2nd law equation for translation for the...

i. ...M mass

ii. ...$3M$ mass



D. Write the Newton's 2nd law equation for rotation for the pulley. Note that TWO of the four forces in your FBD will (nicely!) NOT appear in the equation.

E. Write the appropriate bridge equation.

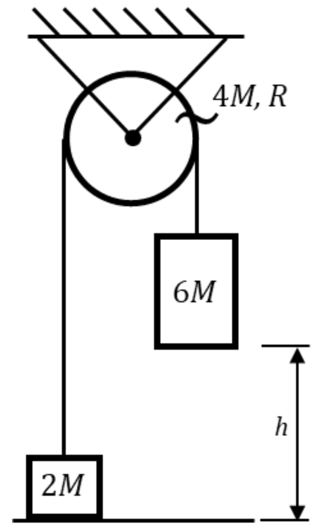
F. In Parts C-E, you have four equations and four unknowns. Do any algebraic work necessary to obtain expressions (in terms of given quantities and fundamental constants) for all four unknown quantities.

Uγ, HW4, P2
Reference Video: “Rotational Kinetic Energy”
YouTube, lasseviren1, ROTATIONAL MOTION playlist

TRANSLATION ROTATION

A. At right, write equations for kinetic energy.

Consider the figure, where the system is released from rest. Your ultimate goal is to derive an expression for the speed with which the $6M$ mass hits the ground, taking into account that the pulley DOES have mass and that there IS friction between the pulley and the rope (but – significantly – NOT any friction in the axle).



B. But first, let’s determine the speed with which the $6M$ mass would hit the ground if there were NO friction. Document the energies using the chart below.

IMPORTANT: We are pretending that there’s NO friction in the pulley or axle.	At release of the system, from rest	At the instant the $6M$ mass hits the ground
U_{grav}		
K_{trans}		
Total E		

C. Use the bottom line of your chart above to determine the speed v_f.

D. Okay, but what if there IS friction between the pulley and the rope? (Still NOT the axle, though...) This will cause the pulley to rotate, and THAT means there will be another term in our conservation of mechanical energy table. Complete the table below. (Recall, also, that we consider our pulley as a disk.)

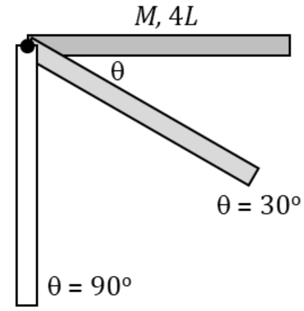
IMPORTANT: Now, there’s friction between the pulley and rope, but still none in the axle.	At release of the system, from rest	At the instant the $6M$ mass hits the ground
U_{grav}		
K_{trans}		
K_{rot}		
Total E		

E. Again, the bottom line of the chart will give you the speed v_f BUT, at the moment, you’re stuck, because there are TWO unknowns and you have only one equation. So, you need the appropriate bridge equation. Write that here. (See HW1, P5, Part A.)

F. Now, combine the bottom line of the chart with your Part E answer to obtain an expression for v_f.

G. Explain why the magnitudes of your Parts C and F answers differ. Specifically, state the physics behind why one of them is larger than (or smaller than) the other one.

Uγ, HW4, P3
Reference Videos: (1) "Rotational Kinetic Energy"
(2) "Rotational Kinetic Energy (Part II)"
YouTube, lasseviren1, ROTATIONAL MOTION playlist



A. The figure shows a rod pinned to the wall at its left end; its right end is free to move. The rod is released from rest horizontally and the right end descends. In EACH of the three depictions in the figure, draw a dot showing where the rod's center of mass is. In each depiction, label this dot "*com*". (Again, ha! "dot-com"...!)

B. Initially, the rod has only ________________ __________ energy. As the rod descends, the amount of this initial energy will decrease. Because the rod's mass is constant – and because gravity is constant – the decrease in this initial energy is due ONLY to the change-in-elevation of the point on the rod abbreviated as _____. The initial type of energy is converted into ______________ ________ energy of the rod, which reaches a maximum when the rod is.....WHERE? ____________________

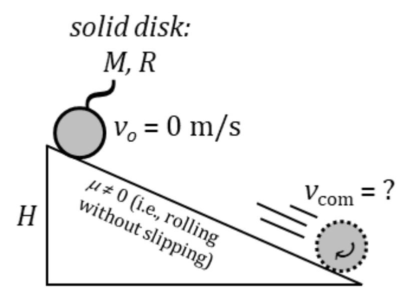
C. Determine the moment of inertia of the rod shown in the figure. (It might be helpful to refer back to HW3, P4, Part I.)

D. Use conservation of mechanical energy to determine the rod's angular speed.
NOTE: Because the axis is NOT moving, you SHALL NOT ("Pass!" Ha, Gandalf...) use a ½ mv_{com}^2 term.

i. for θ = 30°

ii. for θ = 90°

In this next problem, the axis of rotation (i.e., the center of the uniform disk) WILL be moving. Therefore, we DO need a K_{trans} = ½ mv_{com}^2 term, in addition to the K_{rot} = ½ $I\omega^2$ term due to the disk rotating, and the total kinetic energy of the rolling disk will be the SUM of these two types of kinetic energy. But, one more thing, before we get into this problem...



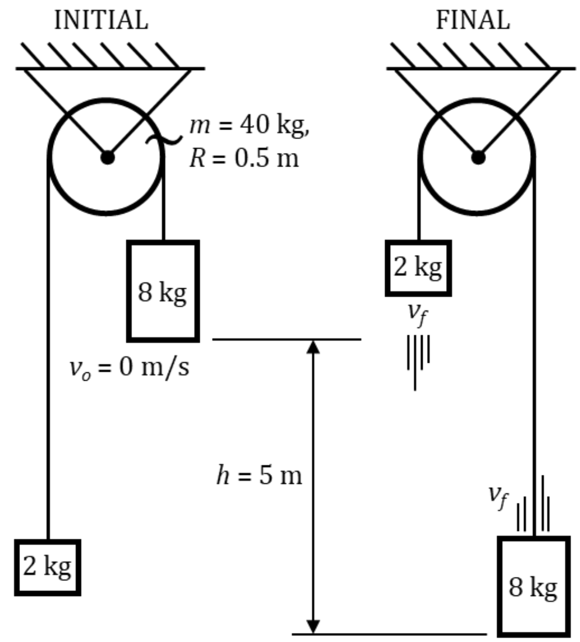
E. Suppose we have a wheel rolling along; it has a certain ω and a certain v_{com}. Suppose also that we have an identical wheel with the same ω, but it isn't rolling; it's just spinning about its central axis and so ITS EDGES have a particular v_{trans}. From the video, how do these v_{com} and v_{trans} values compare to each other?

For a rolling thing, the truth of your Part E answer allows you to use a bridge equation to combine its K_{trans} = ½ mv_{com}^2 term and its K_{rot} = ½ $I\omega^2$ term into a SINGLE quantity. ☺

F. Use the conservation of mechanical energy (and a bridge equation!) to derive the expression for the rolling speed of the uniform disk when it reaches the bottom of the ramp.

Uγ, HW4, P4
Reference Video: "Atwood's Machine Problem w/Pulley of Non-Negligible Mass and Friction in Its Axle"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

Here, we will use the figure and solve three different problems. First, we'll review by assuming there is ZERO friction at all. Second, we'll again review, but we'll consider that the pulley has mass and that the rope doesn't slip on the pulley. And finally, we'll tackle the essence of the video above, by adding into the mix the consideration of friction in the axle.



The figure shows the initial and final states. In each case, the system starts from rest and ends after the masses have moved 5 m. Assume the pulley to be a uniform disk. Use g = 10 m/s^2.

A. Assuming no friction at all (i.e., the pulley CANNOT rotate), use conservation of energy to determine v_f. Feel free to refer back to your work on HW4, P2, Parts B and C. Round your answer to three sig figs.

B. Now, assuming there IS friction between the rope and the pulley (i.e., the pulley WILL rotate without slipping), use conservation of energy to determine v_f. Refer back to your work on HW4, P2, Parts D-F. Again, it is important to realize that we are assuming ZERO friction between the axle and the pulley.

C. Finally, we consider that there IS friction between the axle and wheel. Ultimately, we wish to find the amount of energy that has been converted into internal energy, due to this friction. To do this, I will TELL you what v_f is: It is 4 m/s. Firstly, with reference to your Part B answer, why is v_f = 4 m/s a reasonable value for v_f, in this case?

D. Using conservation of energy, follow the example in the video to determine how much energy has been converted into internal energy between the initial and final states of the system shown, for v_f = 4 m/s.

Uγ, HW4, P5
Reference Video: "Angular Momentum"
YouTube, lasseviren1, ROTATIONAL MOTION playlist

TRANSLATION ROTATION

A. At right, write the appropriate equations for kinetic energy.

B. Write the bridge equation for displacement. (If needed, refer back to HW1, P5, Part A.)

C. Show how your Part B answer can be used to justify the claim that a radian is basically...a nothing, i.e., that it is essentially a unitless unit.

D. Keeping in mind your Part C answer, show that the right sides of your Part A equations yield the same **derived unit**, i.e., a unit that is some multiplication/division combination of one or more SI base units.

E. Your Part D answer should suggest that K_{trans} and K_{rot} are additive. If an object is ________________ but not ______________________, then it has only K_{rot} energy; if it is ______________________ but not ________________, then it has only K_{trans} energy. But, if it is ________________ and, at the same time, its _______ is ______________________ (the phenomenon of wheels that are in the process of _____________ is the most common example) then K_{total} will be the _____ of K_{trans} and K_{rot}. You've already shown that you've accepted this fact in having completed three previous assignments: _______, _______, and _______.

To follow up your Part E answers: In some cases, it is a matter of perspective as to whether something is translating or rotating. Consider the Moon: It doesn't rotate on its axis, in the usual sense; it always keeps the same face directed toward Earth. Suppose we know the Moon's mass is M, it is a distance R from Earth's center, and it translates at a speed v...

F. In the space at right, draw a picture depicting the last sentence of the above paragraph.

Now, if our perspective is...

TRANSLATING, but NOT ROTATING	ROTATING, but NOT TRANSLATING
G. Based on your Parts A and F answers, write an equation for the Moon's K_{trans}. (Easy!)	H. Write the equation for the moment of inertia of a point mass M going around an axis that is a distance R away. (See HW2, P5, Part A.)
Your Parts G and J answers should convince you, for a case such as the Moon, we have to choose EITHER translation OR rotation. We'll get the same answer no matter which one we choose, but we can't choose both. (If we do, we'll get an answer that's 2X too big.)	I. Write the bridge equation for velocity.
	J. Combine your Parts A, H, and I answers to yield an equation for K_{rot}, in terms of known quantities.

Momentum and angular momentum are...different. ("Yo' boy's...DIFFent, Miz Gump.")

LINEAR ANGULAR

K. At right, write the equation for linear momentum AND its rotational analog.

L. Show that the right sides of your Part K equations DO NOT yield the same derived unit.

M. Your Part L answers indicate that, ***unlike*** K_{trans} and K_{rot}, $\vec{p}$ and $\vec{L}$ CANNOT be...