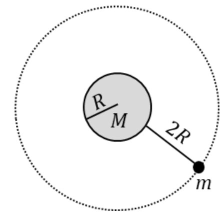
APPC, Mechanics: Unit δ HW 4

Name: ______________________________
Hr: ___ Due at beg of hr on: __________

Uδ, HW4, P1

Reference Videos: (1) "A Quick Review of Planetary Motion"
(2) "A Review of Planetary Motion (Satellite Motion) Part II"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

The figure at right shows a celestial body of mass M and radius R being orbited by a satellite m at an altitude of $2R$ above M's surface. Assume m is in circular orbit.



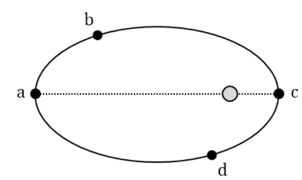
A. Write (and simplify) an expression for the force of gravity on m.

B. Using your Part A answer, apply Newton's 2nd law to derive an expression for the necessary speed v for m to follow a circular path around M.

C. Show that your Part B answer is dimensionally correct, i.e., that the right side's units do, in fact, yield m/s.

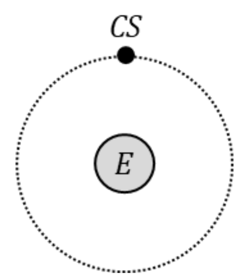
D. Use your Part B answer in deriving an expression for the angular momentum of m.

E. The next figure shows a satellite in elliptical orbit around a celestial body. CIRCLE all correct answers. At which point(s) in the orbit does the satellite have the...?



i. largest force of gravity:	a	b	c	d	all equal to zero	all equal, but NOT zero
ii. largest torque:	a	b	c	d	all equal to zero	all equal, but NOT zero
iii. smallest angular momentum:	a	b	c	d	all equal to zero	all equal, but NOT zero
iv. largest total energy:	a	b	c	d	all equal to zero	all equal, but NOT zero
v. smallest kinetic energy:	a	b	c	d	all equal to zero	all equal, but NOT zero
vi. largest centripetal acceleration:	a	b	c	d	all equal to zero	all equal, but NOT zero
vii. largest tangential acceleration:	a	b	c	d	all equal to zero	all equal, but NOT zero
viii. largest satellite speed:	a	b	c	d	all equal to zero	all equal, but NOT zero

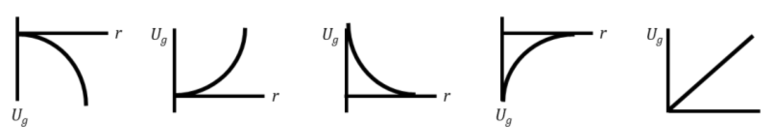
F. The figure shows a communications satellite (CS) in circular orbit around Earth, i.e., the satellite is traveling at the perfect speed for circular orbit. If suddenly the engines on the satellite are fired opposite to the direction of its travel, the satellite will slow down. What simple shape will the new orbit of the satellite have?



G. Based on your Part F answer, draw into the figure an approximate new orbital path.

H. Suppose a planet has a radius equal to 5 Earth radii ($5R_E$) and a mass equal to 125 Earth masses ($125M_E$). (Jupiter has a mass of more than 300 Earth masses, FYI...). In terms of g, determine the gravitational field strength on the surface of this hypothetical planet. Show your work.

I. Which graph exhibits how absolute gravitational potential energy U_g varies with separation r between two masses? CIRCLE your answer.

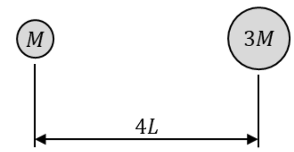


Uδ, HW4, P2

Reference Videos: (1) "A Review of Planetary Motion (Satellite Motion) Part II"
(2) "A Review of Planetary Motion (Satellite Motion) Part III"
YouTube, lasseviren1, PLANETARY/SATELLITE MOTION AND GRAVITY playlist

The figure shows a binary star system that will be revolving around a common center of mass (*com*). Assume both stars follow circular orbits.

A. Use an equation to determine where the system's *com* is. Then, into the figure, put a • where the *com* is, label it, and label the distance from the *com* to each star. HINT: Assume Star M is at the origin.



B. Based on your Part A answers, draw into the figure each star's circular path.

C. How should the orbital periods of Stars M and $3M$ compare?

Let's prove your Part C answer. We'll first find an expression for the period of M...

D. Write and simplify the equation for the force of gravity on M.

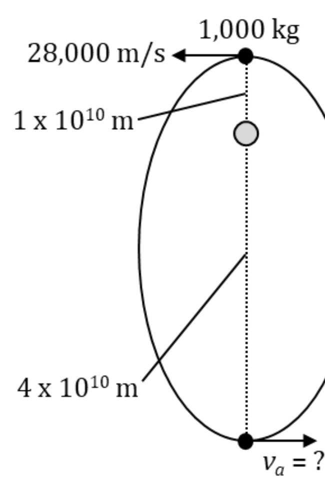
E. Use your Part D answer and Newton's 2nd law to obtain an expression for M's orbital speed v.

F. But speed is distance over time, i.e., $v = \frac{2\pi r}{T}$. Use this equation and your Part E answer to obtain an expression for the period of M.

G. Repeat Parts D-F below, but this time for the $3M$ star.

H. Were you able to verify your Part C answer? (CIRCLE) 😊 😫

The figure at right shows an asteroid in an elliptical orbit around a star. Assume that the asteroid's mass remains constant.



I. For the two points shown in the figure, conservation of angular momentum boils down to the simple equation shown back in Part M of HW1, P5. Use that equation and the information in the figure to determine the speed v_a.

J. Assuming the mass of the star is 7.346×10^{28} kg, now use the conservation of mechanical energy to determine v_a.

K. How do your Parts I and J answers compare?

Uδ, HW4, P3

Reference Videos: (1) "Simple Harmonic Motion Review (Part I)"
(2) "Review of Simple Harmonic Motion (Part II)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

A. A position-time equation for SHM is : $x(t) = 3\cos(\pi t)$. Assume standard SI units. Determine each of the following. On Parts iii through viii, include units. On Parts vii and viii, round to three sig figs.

i. velocity-time equation

ii. acceleration-time equation

iii. amplitude of the motion

iv. angular frequency of the motion

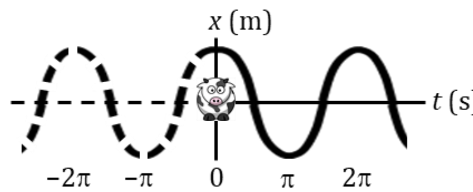
v. frequency of the motion

vi. period of the motion

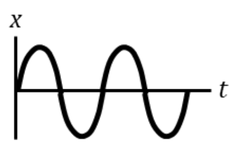
vii. maximum speed

viii. maximum magnitude of acceleration

We now deal with finding the **<u>phase shift</u>** ϕ from the graph of a system oscillating in SHM. You already know that the general form of the displacement equation for such a system is $x(t) = A\cos(\omega t + \varphi)$. If the given graph looks like the right side of the one at right (the non-existent, negative-time part of the curve is shown in dashes), then we know that $\phi = 0$. If the graph DOESN'T look like that, then ϕ is NOT zero. So then, what IS ϕ? Here's what you do: ***<u>MOO</u>ve the Cosine Cow along the time axis of the figure at right until her new location corresponds to t = 0 for the graph you are given.*** If you moved her to the left, then ϕ is minus-something; if you moved her to the right, then ϕ is plus-something. For example, take the graph at right:

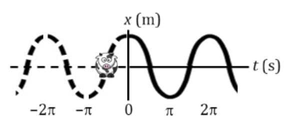


"<u>MOO</u>ve the Cosine Cow; She'll graze to find the phase!"



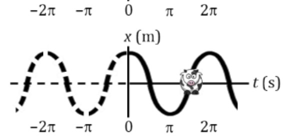
<u>EITHER form of the equation is valid, as they are graphically identical</u>.

MOOving the Cosine Cow to the left yields:



Thus, ϕ is (−) $\pi/2$, and our equation is: $x(t) = A\cos(\omega t - \frac{\pi}{2})$.

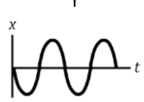
MOOving the Cosine Cow to the right yields:

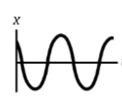


Thus, ϕ is (+) $3\pi/2$, and our equation is: $x(t) = A\cos(\omega t + \frac{3\pi}{2})$.

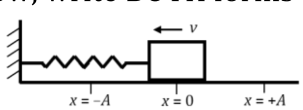
B. Write both the −ϕ and +ϕ forms of the displacement-time equation, as shown above, for each graph below. <u>NOTE</u>: For simplicity, assume that all ϕs are even multiples of ¼ π.

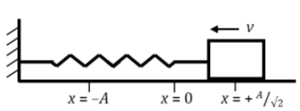
i. 

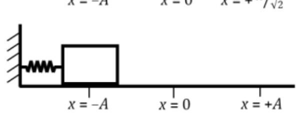
ii. 

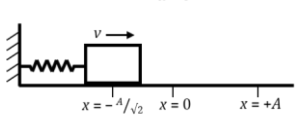
iii. 

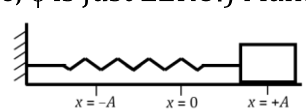
C. *Now, relate equations/graphs/real-life scenarios...* Assuming the mass starts (i.e., $t = 0$) at each location below, write BOTH forms that ϕ could take. (In one case, ϕ is just ZERO.) Make any ϕ a multiple of ¼ π.

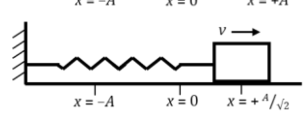
i. 

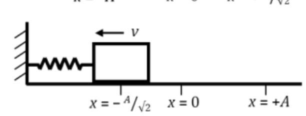
ii. 

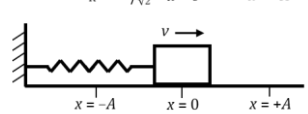
iii. 

iv. 

v. 

vi. 

vii. 

viii. 

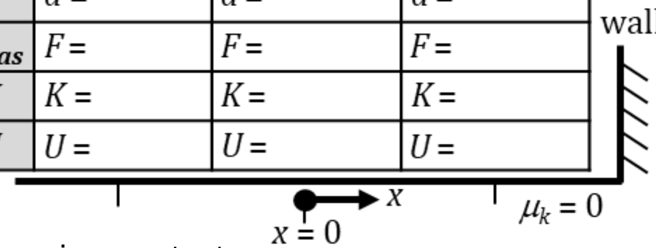
Uδ, HW4, P4

Reference Videos: (1) “Review of Simple Harmonic Motion (Part III)”
(2) “Review of Simple Harmonic Motion (Part IV)”
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist

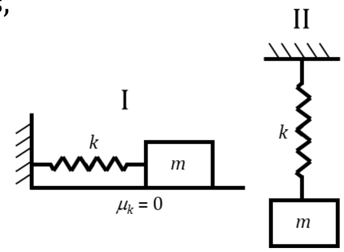
A. The figure shows a wall and a frictionless surface. Although NOT shown in the figure, assume a spring connects the wall and a mass, which oscillates between the tick-marks shown below the surface. Your task is to fill in each of the 18 white boxes to show the value of each quantity at that location. Here are your choices:

zero **(+) max** **(–) max** **(+/–) max**

$\boldsymbol{x}$	x =	x =	x =
$\boldsymbol{v}$	v =	v =	v =
$\boldsymbol{a}$	a =	a =	a =
$\boldsymbol{F_{elas}}$	F =	F =	F =
$\boldsymbol{K}$	K =	K =	K =
$\boldsymbol{U}$	U =	U =	U =



B. The two oscillating systems shown have identical masses, spring constants, and amplitudes. Which system has... (CIRCLE your answers)



i. more energy I II IT’S A TIE

ii. the mass achieving a greater v_{max} I II IT’S A TIE

iii. a greater maximum F_{elas} in the spring I II IT’S A TIE

C. Write the equation for the period of a mass-spring system.

D. Based on your Part C answer, which of the Part B choices has the greater period? I II IT’S A TIE

E. Based on your Part C answer, if we increase the amplitude of either case in Part B, the periods will: INCREASE DECREASE STAY THE SAME

F. Based on your Part C answer, if we set up the Part B cases on the Moon, the periods will: INCREASE DECREASE STAY THE SAME

G. Based on your Part C answer, list two variables that, if changed, will change the period.

H. In Part B’s Case II... Suppose we hook the mass onto the unstretched spring and drop the mass from rest in gravity g, and we want to know the total distance h the mass will fall before the spring stops it.

i. What law or principle of physics should be used to answer this question?

ii. Employ your Part Hi answer to find an expression for the total falling distance h. Show your work.

iii. Show that your Part Hii answer is dimensionally consistent, i.e., that the units on the right side do, in fact, reduce to the proper unit for h.

iv. According to the video, the equilibrium distance (let’s call it e) is equal to _____ h.

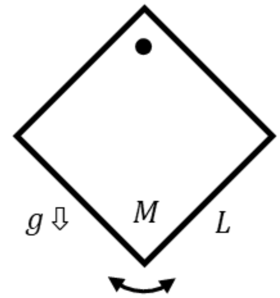
v. Combine your Parts Hii and Hiv answers to obtain an expression for e.

vi. Based on your Part Hv answer, write an expression for the amplitude of the mass’s motion.

vii. Write an expression for the angular frequency of the mass’s motion.

Uδ, HW4, P5

Reference Videos: (1) "Review of Simple Harmonic Motion (Part IV)"
(2) "Review of Simple Harmonic Motion (Part V)"
YouTube, lasseviren1, SIMPLE HARMONIC MOTION playlist



The figure shows a uniform square plate that will rotate on the axis shown in a gravity field g. The square plate has mass M and side length L.

A. Into the figure, draw and label a dot that represents the plate's center of mass.

B. Assume that the distance between the center of mass and the axis is ½ L. Using dimension lines and a two-headed arrow, represent this information in the figure.

C. The rotational inertia *about the com* of any uniform rectangular plate having length l and width b is given by $I_{com} = \frac{1}{12} M\,(l^2 + w^2)$. In terms of M and L, derive an expression for the I_{com} of the plate in the figure.

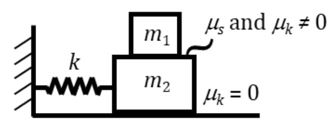
D. But this plate will be rotating about the axis shown, NOT rotating about its *com*.

i. What principle or theorem of physics should be used to address this issue?

ii. Employ the information from Part B as well as your Part Di answer to determine the plate's rotational inertia about the axis shown.

E. This oscillating plate will be what type of pendulum? (CIRCLE) SIMPLE PHYSICAL

F. Based on your Part E answer, employ the proper equation to derive an expression for the period of this pendulum.



G. With reference to the figure, the narrator ultimately derives an expression for the largest displacement x that the system can handle without there being any ____________ between m_1 and m_2. He also makes the important point that the __________ ____________ force between the masses equals zero at the __________________ position. This is because, at that position, the two-mass system is NOT __________________. Therefore, m_1 is also NOT __________________, which means that the ____ _______ on m_1 at that point is ______ ...and the ____ _______ on m_1 at that point (and all others) is, in fact, the __________ ____________ force.

As the last video shows, the displacement satisfying the condition of Part G is $x = \frac{\mu_s g}{k}(m_1 + m_2)$. If we pull the spring farther to the right than x, what will m_2's acceleration be when it reaches location x...?

H. Let's first deal with m_1. Because m_1 will be sliding, there will be a kinetic friction force on it. At right, draw an FBD showing the three forces on m_1. Then, use Newton's 2nd law to find m_1's acceleration a_1.

I. Draw an FBD for m_2 when it is at location x. Show all five forces, two of which are INTERNAL from your Part H answer. (Thus, they are equal and opposite to how they appear in Part H.) Then, write a Newton's 2nd law equation; leave it unsimplified.

J. Substitute the expression for x given above Part H into your Part I answer and solve for a_2.

K. Based on your Parts H and J answers, do both masses have the same acceleration when m_2 is at location x? YES NO